

# The Boer-Mulders effect in unpolarized SIDIS: an analysis of the COMPASS and HERMES data on the $\cos 2\phi$ asymmetry

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We present a phenomenological analysis of the  $\cos 2\phi$  asymmetry recently measured by the COMPASS and HERMES collaborations in unpolarized semi-inclusive deep inelastic scattering. In the kinematical regimes explored by these experiments the asymmetry arises from transverse-spin and intrinsic transverse-momentum effects. We consider the leading-twist contribution, related to the so-called Boer-Mulders transverse-polarization distribution  $h_1^\perp(x, k_T^2)$ , and the twist-4 Cahn contribution, involving unpolarized transverse-momentum distribution functions. We show that a reasonably good fit of the preliminary data sets from COMPASS and HERMES is achieved with a Boer-Mulders function consistent with the main theoretical expectations. Our conclusion is that the COMPASS and HERMES measurements represent the first experimental evidence of the Boer-Mulders effect in SIDIS.

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## I. INTRODUCTION

Among the various observables related to the transverse momentum and the transverse spin of quarks (for reviews, see [1, 2]), the azimuthal asymmetries in unpolarized semi-inclusive deep inelastic scattering (SIDIS) at moderate  $P_T$  have recently attracted a large experimental and theoretical attention (see, for instance, [3–12]). In particular, the  $\cos 2\phi$  asymmetry potentially represents a fundamental source of information on the Boer-Mulders function  $h_1^\perp(x, k_T)$  [13], which measures the transverse polarization asymmetry of quarks inside an unpolarized hadron.

In a previous paper [14], which predated the experimental measurements, we presented a systematic study of the  $\cos 2\phi$  asymmetry in unpolarized SIDIS, taking into account both non perturbative and perturbative contributions. We found that  $\langle \cos 2\phi \rangle$  is of the order of few percent, but our main prediction was that, due to the Boer-Mulders effect, the asymmetry in  $\pi^-$  production should be larger than in  $\pi^+$  production. The recently released COMPASS [15, 16] and HERMES data [17] confirm this expectation (the  $\cos 2\phi$  asymmetry was previously investigated at high  $Q^2$ , where it is dominated by perturbative QCD effects [14], by the EMC [18] and ZEUS [19] experiments).

The purpose of the present work is to perform a phenomenological analysis of the recent  $\langle \cos 2\phi \rangle$  measurements in order to check their mutual compatibility and to extract some information about the Boer-Mulders function. The available data do not allow yet a full fit of  $h_1^\perp$ , with its  $x$  and  $k_T$  dependence: so we will simply relate  $h_1^\perp$  to its chiral-even partner, the Sivers function  $f_{1T}^\perp$  (describing the distribution of unpolarized quarks in a transversely polarized hadron [20, 21]) and limit ourselves to fitting the proportionality coefficient between the two distributions.

We will see that a reasonably good description of both HERMES and COMPASS preliminary data is achieved with a Boer-Mulders function close to that used in Ref. [14] and consistent with various theoretical expectations (impact-parameter approach [22], lattice results [23], large- $N_c$  predictions [24] and model calculations [7, 10–12]). We found however that the quality of the fit depends on the assumptions about the average transverse momenta of quarks for each measurement. One problem emerges also quite clearly: while the  $x$  and  $z$  dependencies of the two sets of data are satisfactorily reproduced, the  $P_T$  behavior of the COMPASS data appears to be incompatible with the HERMES corresponding behavior, and hard to understand phenomenologically.

## II. THE $\cos 2\phi$ ASYMMETRY IN UNPOLARIZED SIDIS

The process we will consider is unpolarized SIDIS:

$$l(\ell) + p(P) \rightarrow l'(\ell') + h(P_h) + X(P_X). \quad (1)$$

The SIDIS cross section is expressed in terms of the invariants

$$x = \frac{Q^2}{2P \cdot q}, \quad y = \frac{P \cdot q}{P \cdot \ell}, \quad z = \frac{P \cdot P_h}{P \cdot q}, \quad (2)$$

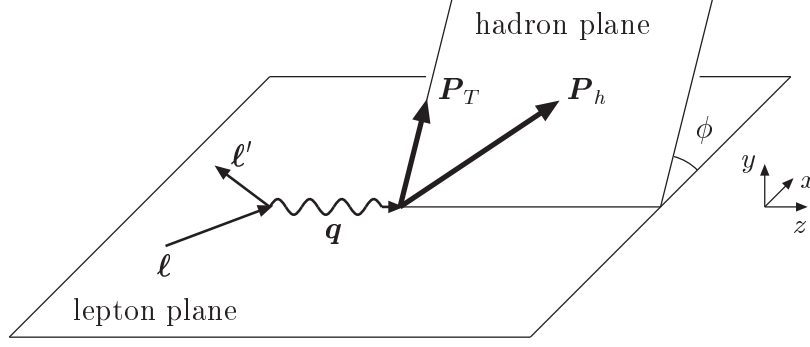


FIG. 1: Lepton and hadron planes in semi-inclusive deep inelastic scattering.

where  $q = \ell - \ell'$  and  $Q^2 \equiv -q^2$ . The reference frame we adopt is such that the virtual photon and the target proton are collinear and directed along the  $z$  axis, with the photon moving in the positive  $z$  direction (Fig. 1). We denote by  $\mathbf{k}_T$  the transverse momentum of the quark inside the proton, by  $\mathbf{P}_T$  the transverse momentum of the hadron  $h$ , and by  $\mathbf{p}_T$  the transverse momentum of  $h$  with respect to the direction of the fragmenting quark. All azimuthal angles are referred to the lepton scattering plane (we call  $\phi$  the azimuthal angle of the hadron  $h$ , see Fig. 1). We follow the conventions of Ref. [25].

We are interested in the low- $P_T$  region,  $P_T \lesssim 1$  GeV. In Ref. [14] we showed that the perturbative contribution to  $\langle \cos 2\phi \rangle$  [26–29] is negligible in this domain. Therefore, in the present analysis we only take into account the non-perturbative effects, related to the transverse momentum of quarks. For small transverse momenta it has been formally proven that SIDIS can be described in terms of Transverse Momentum Dependent (TMD) distribution and fragmentation functions [30]. We will work at tree level, that is in the parton model generalized to include transverse momenta of quarks. At leading twist, there is just one contribution, involving the Boer–Mulders distribution  $h_1^\perp$  coupled to the Collins fragmentation function  $H_1^\perp$  [31], which describes the fragmentation of transversely polarized quarks into polarized hadrons. Concerning higher-twist terms, it is known that there is no twist-3 contribution to  $\langle \cos 2\phi \rangle$  [32], whereas at twist-4 (i.e., at order  $k_T^2/Q^2$ ) the asymmetry receives a contribution from the so-called Cahn term [33, 34], arising from the non-collinear kinematics of quarks.

The  $\phi$ -independent part of the SIDIS differential cross section reads

$$\begin{aligned} \frac{d^5\sigma_{\text{sym}}^{(0)}}{dx dy dz d^2\mathbf{P}_T} &= \frac{2\pi\alpha_{\text{em}}^2 s}{Q^4} \sum_a e_a^2 x [1 + (1-y)^2] \\ &\times \int d^2\mathbf{k}_T \int d^2\mathbf{p}_T \delta^2(\mathbf{P}_T - z\mathbf{k}_T - \mathbf{p}_T) f_1^a(x, k_T^2) D_1^a(z, p_T^2), \end{aligned} \quad (3)$$

where  $f_1^a(x, k_T^2)$  is the unintegrated number density of quarks of flavor  $a$  and  $D_1^a(z, p_T^2)$  is the transverse-momentum dependent fragmentation function of quark  $a$  into the final hadron.

At leading-twist the only  $k_T$ -dependent term contributing to the  $\cos 2\phi$  asymmetry contains the Boer–Mulders distribution  $h_1^\perp$  coupled to the Collins fragmentation function  $H_1^\perp$  of the produced hadron. This contribution to the cross section is given by [13]

$$\begin{aligned} \left. \frac{d^5\sigma_{\text{BM}}^{(0)}}{dx dy dz d^2\mathbf{P}_T} \right|_{\cos 2\phi} &= \frac{4\pi\alpha_{\text{em}}^2 s}{Q^4} \sum_a e_a^2 x (1-y) \\ &\times \int d^2\mathbf{k}_T \int d^2\mathbf{p}_T \delta^2(\mathbf{P}_T - z\mathbf{k}_T - \mathbf{p}_T) \\ &\times \frac{2\mathbf{h} \cdot \mathbf{k}_T \mathbf{h} \cdot \mathbf{p}_T - \mathbf{k}_T \cdot \mathbf{p}_T}{zM_N M_h} h_1^{\perp a}(x, k_T^2) H_1^{\perp a}(z, p_T^2) \cos 2\phi, \end{aligned} \quad (4)$$

where  $M_N$  is the mass of the nucleon,  $M_h$  is the mass of the produced hadron and  $\mathbf{h} \equiv \mathbf{P}_T/P_T$ .

The twist-4 Cahn contribution has the form

$$\left. \frac{d^5\sigma_{\text{C}}^{(0)}}{dx dy dz d^2\mathbf{P}_T} \right|_{\cos 2\phi} = \frac{8\pi\alpha_{\text{em}}^2 s}{Q^4} \sum_a e_a^2 x (1-y)$$

$$\begin{aligned}
& \times \int d^2 \mathbf{k}_T \int d^2 \mathbf{p}_T \delta^2(\mathbf{P}_T - z \mathbf{k}_T - \mathbf{p}_T) \\
& \times \frac{2(\mathbf{k}_T \cdot \mathbf{h})^2 - k_T^2}{Q^2} f_1^a(x, k_T^2) D_1^a(z, p_T^2) \cos 2\phi.
\end{aligned} \tag{5}$$

Notice that the Cahn term is only a part of the total, still unknown, twist-4 contribution.

The asymmetry determined experimentally is defined as

$$A^{\cos 2\phi} \equiv 2 \langle \cos 2\phi \rangle = 2 \frac{\int d\sigma \cos 2\phi}{\int d\sigma}. \tag{6}$$

The integrations are performed over the measured ranges of  $x, y, z$ , with a lower cutoff  $P_T^{\min}$  on  $P_T$ , which represents the minimum value of  $P_T$  of the detected charged particles.

Using the expressions above, the numerator and the denominator of (6) are given by

$$\int d\sigma \cos 2\phi = \frac{4\pi\alpha_{\text{em}}^2 s}{Q^4} \int \int \int \int \sum_a e_a^2 x(1-y) \{ \mathcal{A}[f_1^a, D_1^a] + \frac{1}{2} \mathcal{B}[h_1^{\perp a}, H_1^{\perp a}] \}, \tag{7}$$

$$\int d\sigma = \frac{2\pi\alpha_{\text{em}}^2 s}{Q^4} \int \int \int \int \sum_a e_a^2 x[1 + (1-y)^2] \mathcal{C}[f_1^a, D_1^a], \tag{8}$$

where

$$\int \int \int \int \equiv \int_{P_T^{\min}}^{P_T^{\max}} dP_T P_T \int_{x_1}^{x_2} dx \int_{y_1}^{y_2} dy \int_{z_1}^{z_2} dz \tag{9}$$

and ( $\chi$  is the angle between  $\mathbf{P}_T$  and  $\mathbf{k}_T$ )

$$\begin{aligned}
\mathcal{A}[f_1^a, D_1^a] & \equiv \int d^2 \mathbf{k}_T \int d^2 \mathbf{p}_T \delta^2(\mathbf{P}_T - z \mathbf{k}_T - \mathbf{p}_T) \\
& \times \frac{2(\mathbf{k}_T \cdot \mathbf{h})^2 - k_T^2}{Q^2} f_1^a(x, k_T^2) D_1^a(z, p_T^2) \cos 2\phi \\
& = \int_0^\infty dk_T k_T \int_0^{2\pi} d\chi \frac{2k_T^2 \cos^2 \chi - k_T^2}{Q^2} \\
& \times f_1^a(x, k_T^2) D_1^a(z, |\mathbf{P}_T - z \mathbf{k}_T|^2),
\end{aligned} \tag{10}$$

$$\begin{aligned}
\mathcal{B}[h_1^{\perp a}, H_1^{\perp a}] & \equiv \int d^2 \mathbf{k}_T \int d^2 \mathbf{p}_T \delta^2(\mathbf{P}_T - z \mathbf{k}_T - \mathbf{p}_T) \\
& \times \frac{2 \mathbf{h} \cdot \mathbf{k}_T \mathbf{h} \cdot \mathbf{p}_T - \mathbf{k}_T \cdot \mathbf{p}_T}{z M_N M_h} h_1^{\perp a}(x, k_T^2) H_1^{\perp a}(z, p_T^2) \\
& = \int_0^\infty dk_T k_T \int_0^{2\pi} d\chi \frac{k_T^2 + (P_T/z) k_T \cos \chi - 2k_T^2 \cos^2 \chi}{M M_h} \\
& \times h_1^{\perp a}(x, k_T^2) H_1^{\perp a}(z, |\mathbf{P}_T - z \mathbf{k}_T|^2),
\end{aligned} \tag{11}$$

$$\begin{aligned}
\mathcal{C}[f_1^a, D_1^a] & \equiv \int d^2 \mathbf{k}_T \int d^2 \mathbf{p}_T \delta^2(\mathbf{P}_T - z \mathbf{k}_T - \mathbf{p}_T) f_1^a(x, k_T^2) D_1^a(z, p_T^2) \\
& = \int_0^\infty dk_T k_T \int_0^{2\pi} d\chi f_1^a(x, k_T^2) D_1^a(z, |\mathbf{P}_T - z \mathbf{k}_T|^2).
\end{aligned} \tag{12}$$

### III. PARAMETRIZATIONS OF DISTRIBUTION AND FRAGMENTATION FUNCTIONS

To calculate the azimuthal asymmetries we need first of all the  $k_T$ -dependent unpolarized distribution functions, which we assume to have a Gaussian behavior in  $k_T$ :

$$f_1^q(x, k_T^2) = f_1^q(x) \frac{e^{-k_T^2/\langle k_T^2 \rangle}}{\pi \langle k_T^2 \rangle}. \tag{13}$$

$A_u = -0.35$	$A_d = 0.90$	$A_s = 0.24$
$A_{\bar{u}} = -0.04$	$A_{\bar{d}} = 0.40$	$A_{\bar{s}} = -1$
$\alpha_u = 0.73$	$\alpha_d = 1.08$	$\alpha_{sea} = 0.79$
$\beta = 3.46$	$M_1^2 = 0.34 \text{ (GeV}/c)^2$	

TABLE I: Parameters of the Siverts function used in Eqs. (16,17)

The Gaussian dependence of the transverse-momentum distribution functions is supported by a recent lattice study [35]. The integrated unpolarized distribution functions  $f_1^q$  are taken from the GRV98 fit [36].

The available data on  $\langle \cos 2\phi \rangle$  do not allow a full extraction of the Boer-Mulders function. Thus we simply take  $h_1^\perp$  to be proportional to the Siverts function  $f_{1T}^\perp$ ,

$$h_1^{\perp q}(x, k_T^2) = \lambda_q f_{1T}^{\perp q}(x, k_T^2), \quad (14)$$

with a coefficient  $\lambda_q$  to be fitted to the data. Various theoretical arguments (based on the impact-parameter picture [22], on large- $N_c$  arguments [24], and on model calculations [10–12]) suggest that the  $u$  and  $d$  components of  $h_1^\perp$ , at variance with  $f_{1T}^\perp$ , should have the same sign and in particular be both negative (which means that  $\lambda_d$  should be negative). This is indeed what we find in our analysis. Moreover, the impact-parameter approach [22] combined with lattice results [23] predicts a  $u$  component of  $h_1^\perp$  larger in magnitude than the corresponding component of  $f_{1T}^\perp$ , and the  $d$  components of  $h_1^\perp$  and  $f_{1T}^\perp$  with approximately the same magnitude (and opposite sign).

We parametrize the Boer-Mulders function using the Ansatz (14) and taking the Siverts function from a fit to single-spin asymmetry data [37]. Thus we set

$$h_1^{\perp q}(x, k_T^2) = \lambda_q f_{1T}^{\perp q}(x, k_T^2) = \lambda_q \rho_q(x) \eta(k_T) f_1^q(x, \mathbf{k}_T^2), \quad (15)$$

where

$$\rho_q(x) = A_q x^{a_q} (1-x)^{b_q} \frac{(a_q + b_q)^{(a_q + b_q)}}{a_q^{a_q} b_q^{b_q}}, \quad (16)$$

$$\eta(k_T) = \sqrt{2e} \frac{M_P}{M_1} e^{-k_T^2/M_1^2}. \quad (17)$$

Here  $M_P$  is the proton mass,  $A_q$ ,  $a_q$ ,  $b_q$  and  $M_1$  are parameters determined in [37] (see Table I). Being a quark spin asymmetry,  $f_{1T}^\perp$  must satisfy a positivity bound, which is automatically fulfilled by the parametrization of Ref. [37]. Notice that the Siverts function parametrization, as defined in Ref. [37], is:  $\Delta^N f_q(x, k_\perp) = -2 \frac{k_\perp}{M_P} f_{1T}^{\perp q}(x, k_\perp)$ .

Concerning the antiquark Boer-Mulders distributions, the SIDIS (at least, the present ones) are not able to constrain them. Thus we simply take the Boer-Mulders antiquark distributions to be equal in magnitude to the corresponding Siverts distributions and both negative. Note that the Drell-Yan measurements of the  $\cos 2\phi$  asymmetry [38, 39] would in principle give information about the antiquark sector [40, 41], but most of the present data seem to be explainable in terms of perturbative QCD [42, 43].

Let us now turn to the fragmentation functions. We distinguish their favored and unfavored components, according to the following general relations

$$D_{\pi^+/u} = D_{\pi^+/\bar{d}} = D_{\pi^-/d} = D_{\pi^-/\bar{u}} \equiv D_{\text{fav}} \quad (18)$$

$$D_{\pi^+/d} = D_{\pi^+/\bar{u}} = D_{\pi^-/u} = D_{\pi^-/\bar{d}} = D_{\pi^\pm/s} = D_{\pi^\pm/\bar{s}} \equiv D_{\text{unf}}, \quad (19)$$

The  $p_T$ -dependent unpolarized fragmentation  $D_1(z, p_T^2)$  is assumed to have the form

$$D_1(z, p_T^2) = D_1(z) \frac{e^{-p_T^2/\langle p_T^2 \rangle}}{\pi \langle p_T^2 \rangle}, \quad (20)$$

again with a Gaussian behavior in  $p_T$ . The integrated fragmentation function  $D_1(z)$  is taken from the the DSS fit [44].

Collins fragmentation function	$A_{fav}^C = 0.44$	$N_{unf}^C = -1.00$
	$\gamma = 0.96$	$\delta = 0.01$
	$M_C^2 = 0.91 \text{ (GeV}^2/\text{c)}$	

TABLE II: Parameters of the favored and unfavored Collins fragmentation functions [45].

For the Collins function we use the parametrization of [45], based on a combined analysis of SIDIS and  $e^+e^-$  data:

$$H_1^{\perp q}(z, p_T^2) = \rho_q^C(z) \eta^C(p_T) D_1(z, p_T^2), \quad (21)$$

with

$$\rho_q^C(z) = A_q^C z^\gamma (1-z)^\delta \frac{(\gamma + \delta)^{(\gamma + \delta)}}{\gamma^\gamma \delta^\delta} \quad (22)$$

$$\eta^C(p_T) = \sqrt{2e} \frac{zM_h}{M_C} e^{-p_T^2/M_C^2}, \quad (23)$$

We let the coefficients  $A_q^C$  to be flavor dependent ( $q = u, d$ ), while all the exponents  $\gamma, \delta$  and the dimensional parameter  $M_C$  are taken to be flavor independent. The parametrization is devised in such a way that the Collins function satisfies the positivity bound (remember that  $H_1^\perp$  is essentially a transverse momentum asymmetry). The values of the parameters as determined in the fit of Ref. [45] are listed in Table II.

The remaining crucial ingredients to be considered are the average values of  $k_T^2$  and  $p_T^2$ . Notice that the following kinematical relation holds between the transverse momentum of the produced hadron and the transverse momenta of quarks:

$$\langle P_T^2 \rangle = \langle p_T^2 \rangle + z^2 \langle k_T^2 \rangle. \quad (24)$$

Due to the limitations of the present data sets it is not possible to treat  $\langle k_T^2 \rangle$  and  $\langle p_T^2 \rangle$  as two additional parameters to be determined by the fit. We have to make some assumptions about them.

In our main fit (hereafter called Fit 1) we take  $\langle k_T^2 \rangle$  and  $\langle p_T^2 \rangle$  from the analysis of the azimuthal dependence of the unpolarized SIDIS cross section performed in Ref. [5]:

$$\langle k_T^2 \rangle = 0.25 \text{ GeV}^2, \quad \langle p_T^2 \rangle = 0.20 \text{ GeV}^2. \quad (25)$$

We also tried another fit (“Fit 2”), using for HERMES the values of  $\langle k_T^2 \rangle$  and  $\langle p_T^2 \rangle$  given by their own analysis of the  $P_T$  spectrum, which turns out to be reproduced by Monte Carlo calculations [46] with  $\langle k_T^2 \rangle = 0.18 \text{ GeV}^2$  and a  $z$ -dependent transverse momentum of the fragmenting quark,  $\langle p_T^2 \rangle = 0.42 z^{0.37} (1-z)^{0.54} \text{ GeV}^2$ . In the  $z$  range of interest this is very well approximated by  $\langle p_T^2 \rangle \simeq 0.20 \text{ GeV}^2$ . Thus in our Fit 2 we choose for HERMES:

$$\langle k_T^2 \rangle = 0.18 \text{ GeV}^2, \quad \langle p_T^2 \rangle = 0.20 \text{ GeV}^2. \quad (26)$$

We have no similar information for the COMPASS measurement and for their data we still use in Fit 2 the values (25). Therefore Fit 2 is characterized by a  $\langle k_T^2 \rangle$  which is different for the two sets of data. As we shall see in the next Section, Fit 2 turns out to be significantly better than Fit 1.

Finally, concerning a possible flavor-dependence of  $\langle k_T^2 \rangle$  [47], we showed in our previous paper [14] that it hardly affects the results, hence we shall not take it into account here (it should also be remarked that the experimental evidence for a flavor-dependent  $\langle k_T^2 \rangle$  is still far from being established).

In summary the assumptions of our fits are:

- $h_1^{\perp q}(x, k_T^2) = \lambda_q f_{1T}^{\perp q}(x, k_T^2)$  for  $u$  and  $d$  quarks while  $h_1^{\perp \bar{q}}(x, k_T^2) = -|f_{1T}^{\perp \bar{q}}(x, k_T^2)|$  for sea quarks, with  $f_{1T}^{\perp q, \bar{q}}$  functions as given in Ref. [37].
- The Collins functions  $H_1^{\perp q}(z, p_T)$  is taken as in Ref. [45].
- Gaussian transverse momentum distribution is assumed for all the distribution/fragmentation functions.

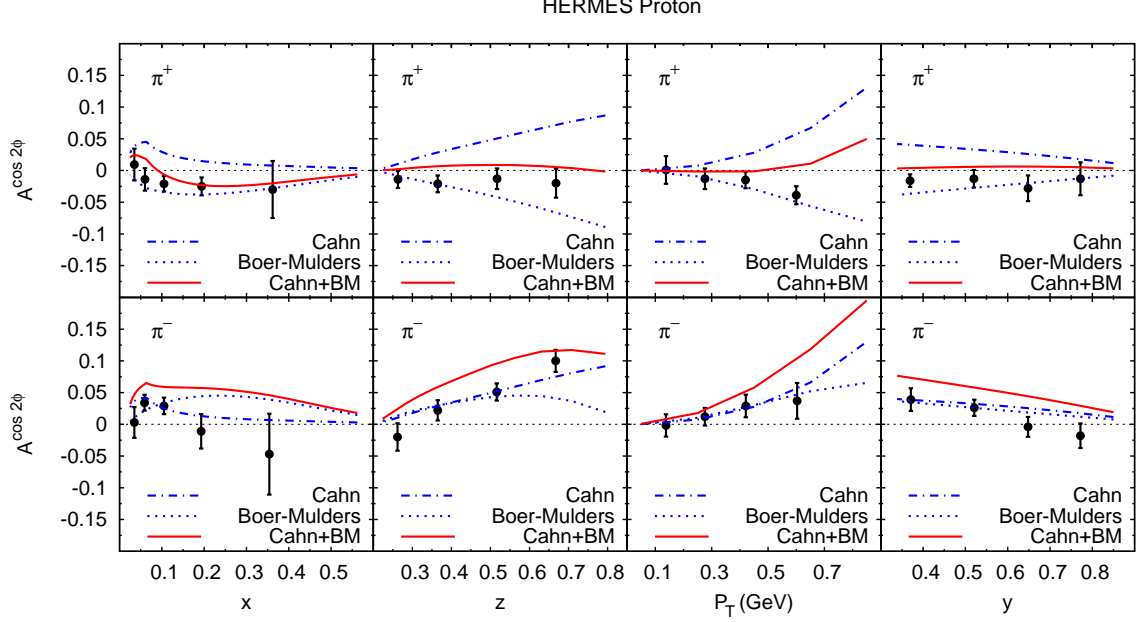


FIG. 2: Our Fit 1 to the HERMES preliminary proton-target data [17]. The dot-dashed line is the Cahn contribution, the dotted line is the Boer-Mulders contribution, the continuous line is the resulting asymmetry taking both contributions into account.

- In particular the average transverse momenta for the unpolarized distribution and fragmentation functions are respectively:

Fit 1:  $\langle k_T^2 \rangle = 0.25 \text{ GeV}^2$ ,  $\langle p_T^2 \rangle = 0.20 \text{ GeV}^2$  for both HERMES and COMPASS data.

Fit 2:  $\langle k_T^2 \rangle = 0.25 \text{ GeV}^2$ ,  $\langle p_T^2 \rangle = 0.20 \text{ GeV}^2$  for COMPASS data;  
 $\langle k_T^2 \rangle = 0.18 \text{ GeV}^2$ ,  $\langle p_T^2 \rangle = 0.20 \text{ GeV}^2$  for HERMES data.

#### IV. ANALYSIS OF THE $\langle \cos 2\phi \rangle$ DATA

Data on the  $\cos 2\phi$  asymmetry in unpolarized SIDIS at small  $P_T$  have been recently presented in a preliminary form by both the HERMES Collaboration [17] and by COMPASS [15, 16] (while this analysis was in progress the CLAS Collaboration at JLab has released some results on  $\langle \cos 2\phi \rangle$ , but their conclusion is that the precision of the data does not allow obtaining information about the Boer-Mulders function [48]). The first qualitative evidence coming from both COMPASS and HERMES measurements is a larger asymmetry for  $\pi^-$  production compared to  $\pi^+$ . This difference was predicted in Ref. [14] to be a signature of the Boer-Mulders effect, which has opposite signs for  $\pi^+$  and  $\pi^-$  (whereas the Cahn contribution, is the same for  $\pi^+$  and  $\pi^-$ ).

Fitting the HERMES and COMPASS data as explained in the previous Section we find in Fit 1 the following values for the coefficients  $\lambda_u$  and  $\lambda_d$ :

$$\lambda_u = 2.0 \pm 0.1, \quad \lambda_d = -1.111 \pm 0.001 \quad (\text{Fit 1}) \quad (27)$$

This implies that  $h_1^{\perp u}$  and  $h_1^{\perp d}$  are both negative. The  $\chi^2$  per degree of freedom of Fit 1 is  $\chi^2/d.o.f. = 3.73$ . The value of  $\lambda_d$  corresponds to the saturation of the positivity bound of  $h_1^{\perp d}$ . Errors on the parameters were calculated with  $\Delta\chi^2 = 1$ .

Notice that we have excluded from the fit the COMPASS data in  $P_T$ , which – as we will see below – are clearly incompatible with the HERMES  $P_T$  data and have a counter-intuitive behavior (it is in fact difficult to envisage a transverse-momentum dependence of distribution and fragmentation functions able to describe them).

The first moments (in  $k_T^2$ ) of the Boer-Mulders distributions  $h_1^{\perp u}$  and  $h_1^{\perp d}$ ,

$$h_1^{\perp(1)}(x) \equiv \int d^2\mathbf{k}_T \frac{k_T^2}{2M^2} h_1^{\perp}(x, k_T^2), \quad (28)$$

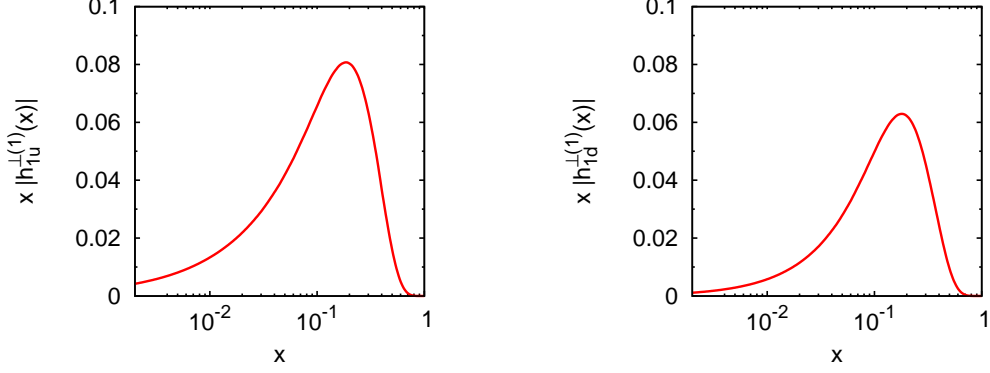


FIG. 3: The first moments of  $h_1^{\perp u}$  and  $h_1^{\perp d}$  from Fit 1.

are displayed in Fig. 3.

In Fig. 2 we plot  $A^{\cos 2\phi} \equiv 2 \langle \cos 2\phi \rangle$  for  $\pi^+$  and  $\pi^-$  production at HERMES with a proton target, with the results of Fit 1. The asymmetry is shown as a function of one variable at a time,  $x$ ,  $z$  and  $P_T$ ; the integration over the unobserved variables has been performed over the measured ranges of the HERMES experiment,

$$\begin{aligned} Q^2 > 1 \text{ GeV}^2, \quad W^2 > 10 \text{ GeV}^2, \quad P_T > 0.05 \text{ GeV} \\ 0.023 < x < 1.0, \quad 0.2 < z < 1.0, \quad 0.3 < y < 0.85 \\ 0.2 < x_F < 1. \end{aligned} \quad (29)$$

Note that the Boer-Mulders contributions to  $\pi^+$  and  $\pi^-$  production are opposite in sign. In fact, we have

$$\begin{aligned} \langle \cos 2\phi \rangle_{\text{BM}}^{\pi^+} &\sim e_u^2 h_1^{\perp u}(x) H_1^{\perp \text{fav}}(z) + e_d^2 h_1^{\perp d}(x) H_1^{\perp \text{unf}}(z), \\ \langle \cos 2\phi \rangle_{\text{BM}}^{\pi^-} &\sim e_u^2 h_1^{\perp u}(x) H_1^{\perp \text{unf}}(z) + e_d^2 h_1^{\perp d}(x) H_1^{\perp \text{fav}}(z), \end{aligned} \quad (30)$$

and, as far as  $H_1^{\perp \text{unf}}(z) \simeq -H_1^{\perp \text{fav}}(z)$  [49], one gets different signs for the Boer-Mulders effect for positive and negative pions. The combination of the Boer-Mulders term with the Cahn term, which is positive and exactly the same for  $\pi^+$  and  $\pi^-$  (if the  $k_T$ -dependence of the distributions is flavor blind) gives a resulting asymmetry which is larger for  $\pi^-$  than for  $\pi^+$ .

Fig. 4 shows our Fit 1 to  $A^{\cos 2\phi_h}$  at HERMES with a deuteron target. We have neglected nuclear corrections and used isospin symmetry to relate the distribution functions of the neutron to those of the proton.

The experimental cuts of the COMPASS experiment (which runs with a deuteron target) are:

$$Q^2 \geq 1 \text{ GeV}^2, \quad W^2 > 25 \text{ GeV}^2, \quad (31)$$

$$0.2 < z < 0.85, \quad 0.1 \leq y \leq 0.9 \quad (32)$$

$$P_T > 0.1 \text{ GeV}. \quad (32)$$

In Fig. 5 we show our fit to the COMPASS data. One clearly sees that the  $P_T$  dependence of these data is incompatible with the HERMES one and hard to understand theoretically.

Let us now come to Fit 2. In this case, the coefficients  $\lambda_u$  and  $\lambda_d$  are found to be

$$\lambda_u = 2.1 \pm 0.1, \quad \lambda_d = -1.111 \pm 0.001 \quad (\text{Fit 2}), \quad (33)$$

and are very close to those of Fit 1 (again,  $h_1^{\perp d}$  saturates its positivity bound). Thus, the  $x$ -dependence of the Boer-Mulders functions is essentially the same in the two fits. However, the  $\chi^2$  per degree of freedom of Fit 2 is significantly smaller:  $\chi^2/d.o.f. = 2.41$ . In Figs. 6, 7 and 8 we show the results of Fit 2 for  $A^{\cos 2\phi}$  compared to the data.

In both Fit 1 and Fit 2 we used only statistical and, when provided (HERMES), systematic uncertainties of the experimental data. Given the quality of present data and the theoretical uncertainties related to the twist four contributions, we did not attempt a more sophisticated analysis of the  $x$  dependence of Boer-Mulders functions and used the simplified assumption of Eq. 14.

The main difference between the two fits resides in the Cahn term, which is strongly sensitive to the average value of  $k_T^2$ . The fact that the data prefer the fit with  $\langle k_T^2 \rangle_{\text{HERMES}} \neq \langle k_T^2 \rangle_{\text{COMPASS}}$  seems to indicate that the twist-4

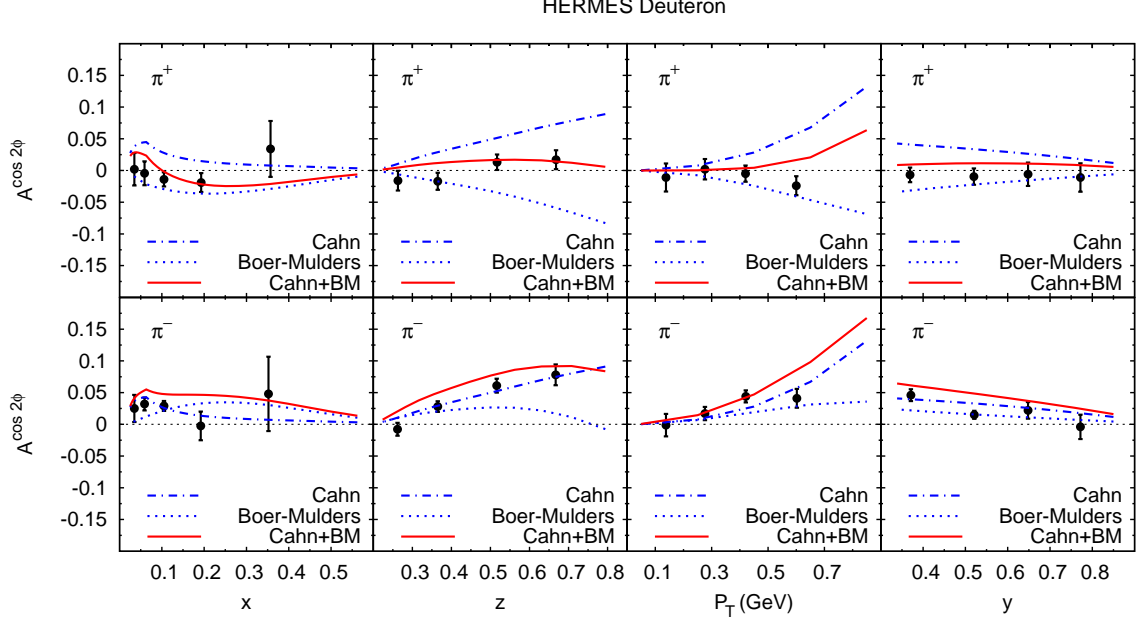


FIG. 4: Our Fit 1 to the HERMES preliminary deuteron-target data. The line labels are the same as in Fig. 2.

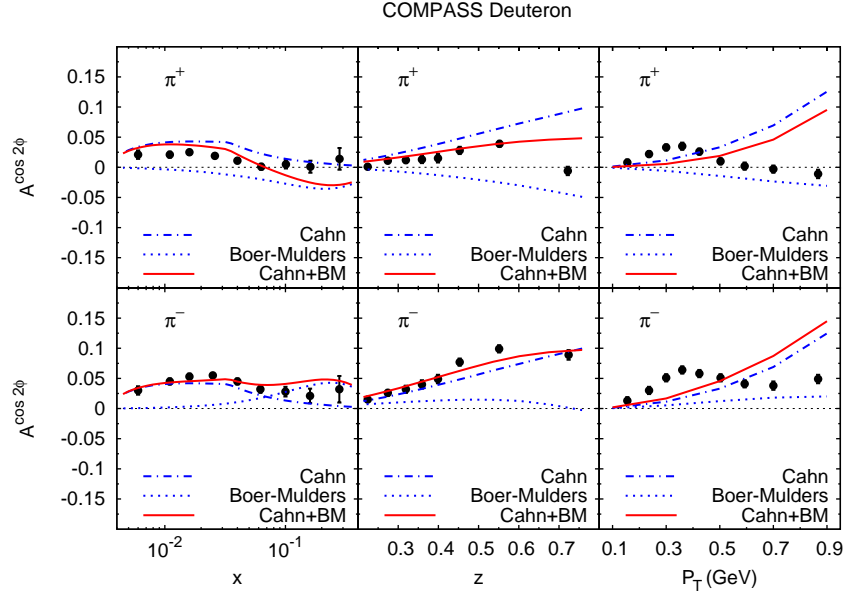


FIG. 5: Our Fit 1 to the COMPASS preliminary data (deuteron target) [15, 16]. The line labels are the same as in Fig. 2.

contributions are different in the kinematics of the two experiments. This is a further indication of the fact that the  $\cos 2\phi$  fit is strongly affected by twist-4 contributions, which are not yet fully known.

Our analysis shows that, as far as the  $x$  and  $z$  dependencies are concerned, both the HERMES and COMPASS data are fairly well described. The resulting Boer-Mulders distributions of quarks have the expected sign [10, 24, 50]. Moreover, looking at eq. (27) or (33), one sees that the Boer-Mulders  $u$  distribution is larger by a factor 2 compared to the  $u$  Sivers distribution, whereas the Boer-Mulders and Sivers  $d$  distributions have approximately the same magnitude. This is in agreement with the predictions of the impact-parameter approach [22] combined with lattice



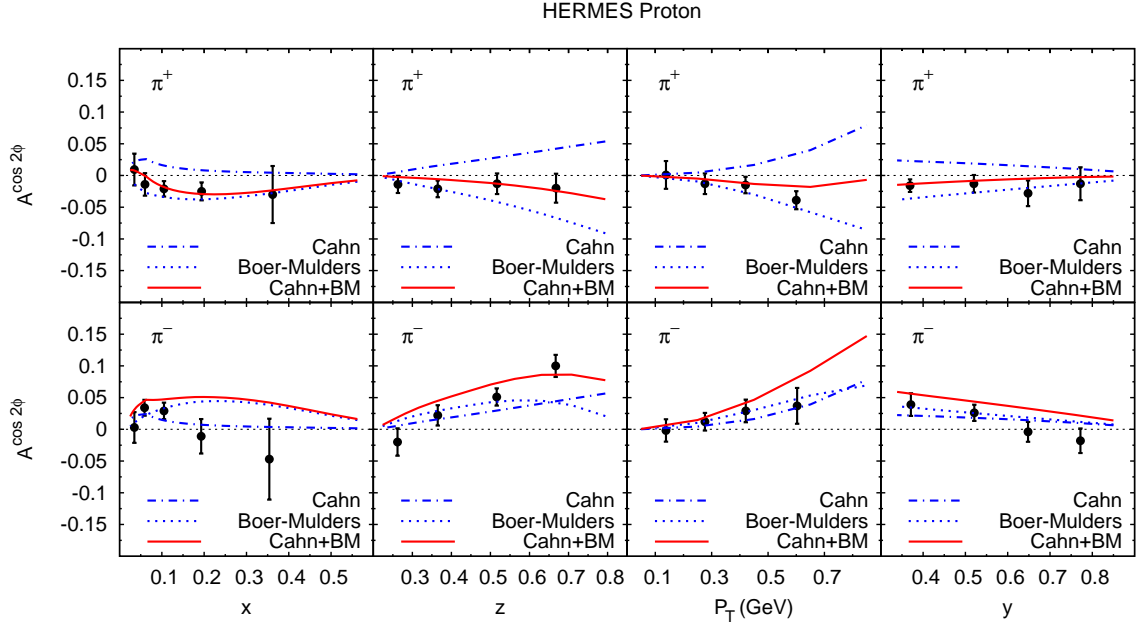


FIG. 6: Our Fit 2 to the HERMES preliminary proton data. The line labels are the same as in Fig. 2.

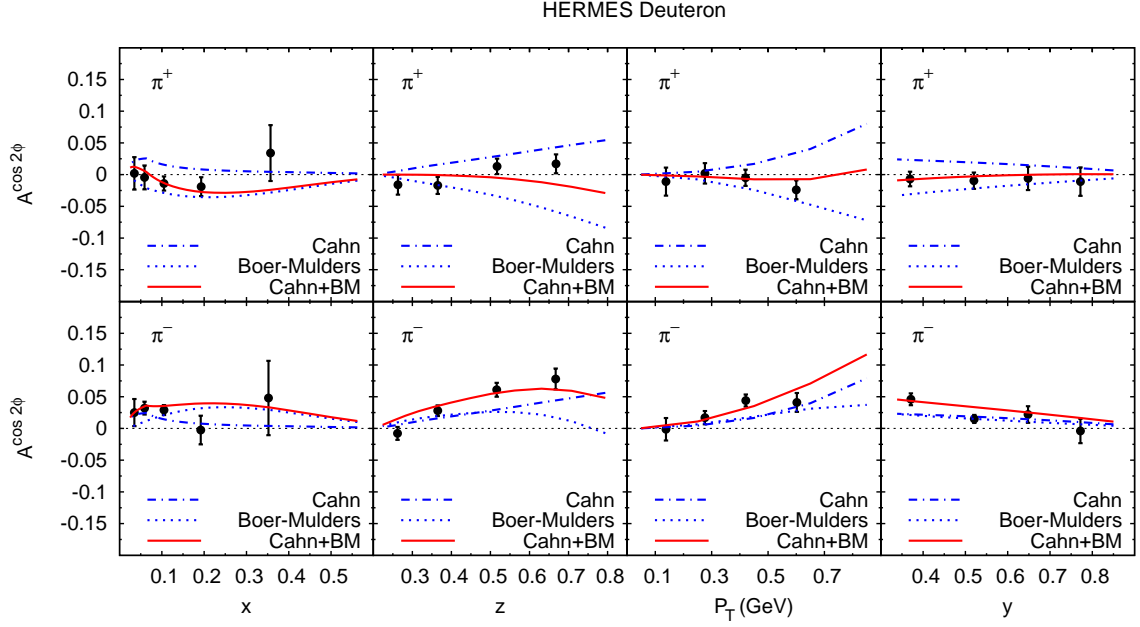


FIG. 7: Our Fit 2 to the HERMES preliminary deuteron target data. The line labels are the same as in Fig. 2.

results [23].

## V. CONCLUSIONS AND PERSPECTIVES

The present unpolarized SIDIS data on azimuthal  $\cos 2\phi$  asymmetries, although still preliminaries, represent a clear manifestation of the Boer-Mulders effect. However, they are not sufficient to allow a full extraction of  $h_1^\perp$ . In

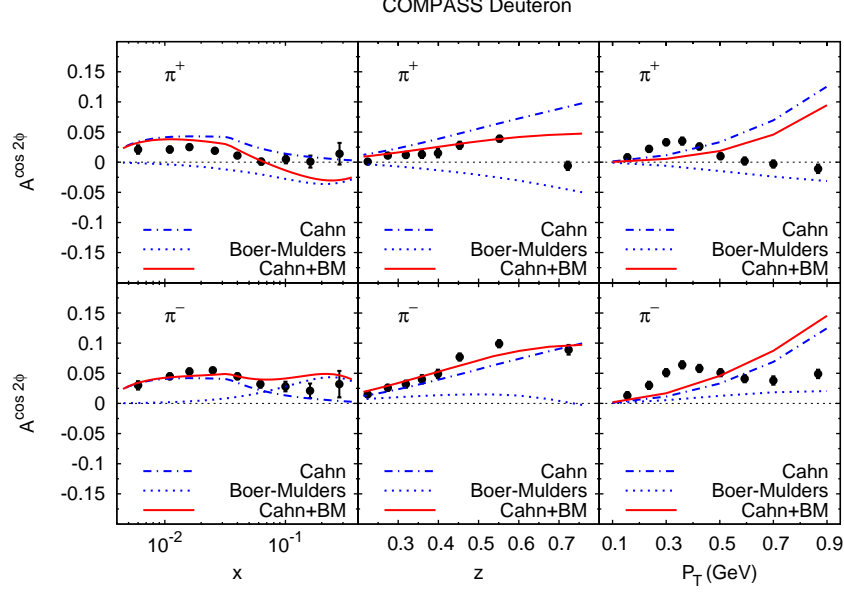


FIG. 8: Our Fit 2 to the COMPASS preliminary data. The line labels are the same as in Fig. 2.

particular, twist-4 Cahn contribution turns out to be comparable in size to twist-2 Boer-Mulders contribution so measurements of the  $\cos 2\phi$  asymmetry at different values of  $Q^2$  are desirable. Measurements at proposed Electron Ion Collider [51, 52] at higher  $Q^2$  would allow separation of pure twist-2 from higher twist contributions. To minimize the influence of Cahn contribution which is supposed to be flavor blind one can measure  $\cos 2\phi$  asymmetry of the difference  $\pi^- - \pi^+$ . Moreover, the antiquark Boer-Mulders distributions turn out to be essentially unconstrained. Therefore, it would be important not only to have more SIDIS data (the JLab experiments will play in the next future a very important role in this respect), but also to explore other processes, like Drell-Yan (DY) production. The kinematics of the recent E866/NuSea DY data [38, 39] is such that they are partly dominated by perturbative effects. An attempt to extract  $h_1^\perp$  from these data has been made in Refs. [40, 41]. The first moments of quark Boer-Mulders functions found in these analysis agree with our results in the relative sign of  $u$  and  $d$  quark Boer-Mulders functions. The magnitudes cannot be easily compared. First of all, as we said, the extraction of  $h_1^\perp$  from E866/NuSea data is affected by perturbative effects which in Refs. [40, 41], have not been taken into account even at large  $q_T$ . Second in  $pp$  and  $pD$  DY processes it is possible to extract only the products of quark-antiquark Boer-Mulders functions. In order to separate them, in Refs. [40, 41] the positivity bound has been employed, obtaining an allowed range for each distribution. However one should add to this range the statistical errors of the fit, which results in a very large final uncertainty on the magnitude of  $h_1^\perp$ .

The  $pp$  or  $pD$   $\cos 2\phi$  DY asymmetry, which involves the sea distributions, is very small. A larger asymmetry is predicted for  $p\bar{p}$  DY production [53–57], a process to be studied in the next years at the GSI High-Energy Storage Ring [58–60]. Another reaction probing valence distributions is  $\pi N$  DY, under investigation by the COMPASS collaboration in their hadronic program [61].

A combined analysis of all these data will represent a decisive step towards a better knowledge of the Boer-Mulders function and of the phenomena originating from it.

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